Time Value of Money

Lesson 12

KEY CONCEPTS

■ Time Value of Money ■ Compound and Simple Interest ■ Present Value ■ Future Value ■ Annuity

Learning Objectives

To understand:

- Meaning and significance of time value of money.
- Future value of a single present cash flow.
- Future value of a series of unequal cash flows over a period of time.
- Future value of a series of equal cash flows over a period of time (FV of an annuity)

Lesson Outline

- Introduction
- Concepts of Time Value of Money
- Present Value of an Uneven Series
- Present Value of an Annuity
- Doubling Period
- Lesson Round-Up
- Glossary
- Test Yourself
- List of Further Readings
- Other References

Introduction

Money has time value. A rupee today is more valuable than a rupee a year hence. The key reasons or it are as under:

i) Individuals, in general, prefer current consumption to future consumption.

ii) Capital can be employed productively to generate positive returns. An investment of one rupee today would grow to (1+r) a year hence (r is the rate of return earned on the investment).

iii) In an inflationary period a rupee today represents a greater rea purchasing power than a rupee a year hence.

Most financial problems involve cash flows occurring at different points of time. These cash flows have to be brought to the same point of time for purposes of comparison and aggregation. Hence one should understand the tools of compounding and discounting which underlie most of what we do in finance- from valuing securities to analysing projects, from determining lease rentals to choosing the right financing instruments, from setting up the loan amortisation schedules to valuing companies, so on and so forth.

Time Value of Money (TVM) is a fundamental financial concept, stating that the current value of money is higher than its future value, given its potential to earn in the years to come. Thus, it suggests that a sum of money in hand is greater in value than the same sum of money received in the next couple of years.

'Time value of money' is central to the concept of finance. It recognizes that the value of money is different at different points of time. Since money can be put to productive use, its value is different depending upon when it is received or paid.

Key takeaways-

1. The time value of money means that a sum of money is worth more now than the same sum of money in the future.
2. The principle of the time value of money means that it can grow only through investing so a delayed investment is a lost opportunity.
3. The formula for computing the time value of money considers the amount of money, its future value, the amount it can earn, and the time frame.
4. For savings accounts, the number of compounding periods is an important determinant as well.
5. Inflation has a negative impact on the time value of money because your purchasing power decreases as prices rise.

CONCEPTS OF TIME VALUE OF MONEY

Compound and Simple Interest

So far we have observed the cases where money is invested at compound interest which means that each interest payment is reinvested to earn further interest in future periods. By contrast, if no interest is earned on interest the investment earns only simple interest. In such a case the investment grows as follows:

Future value = Present Value [1+Number of years x Interest rate].

For example, an investment of Rs.1,000, if invested at 12 percent simple interest rate will in 5 years time become:

1,000 [1 + 5 x 0.12] = Rs.1,600.

The following exhibit 1 shows how an investment of Rs.1,000 grows over time under simple interest as well as compound interest when the interest rate is 12 percent. From this exhibit one can comprehend the power of compound interest. As Albert Einstein once remarked: "I don't know what the seven of the world are, but I know the eight- compound interest".

Exhibit 1

Power of Compounding

Year	*Simple Interest*			*Compound Interest*		
	Starting Balance + Interest =Ending Balance			*Starting Balance + Interest = Ending Balance*		
	Starting Balance	*Interest*	*Ending Balance*	*Starting Balance*	*Interest*	*Ending Balance*
1	1000	100	1100	1000	100	1100
5	1400	100	1500	1464	146	1610
10	1900	100	2000	2358	236	2594
20	2900	100	3000	6116	612	6728
50	5900	100	6000	106,718	10672	117,390
100	10900	100	11000	12,527,829	1,252,783	13,780,612

Thus, from the above exhibit it can be observed that how money grows under simple interest and compound interest. Note that under simple interest the growth is linear and under compound interest the growth is exponential.

In the ensuing paragraphs now we will focus on the crucial concepts of time value of money, i.e., Present value of a single amount, Future value of a single amount.

1. Present Value of a Single Amount

Many times in business and life, we want to determine the value today of receiving a specific single amount at some time in the future. For example, suppose you want to know the value today of receiving $15,000 at the end of 5 years if a rate of return of 12% is earned.

Another way of asking this question is: What amount would you need to invest today at 12% compounded annually in order to receive $15,000 after 5 years?

Problems and questions like this are known as "present value of a single amount problems." This is because we are interested in finding the present value, or the value today, of receiving a set sum in the future. Intuitively, we know that the present value will be less than the future value. For example, if you had the choice of receiving $12,000 today or in 2 years, you would take the $12,000 today.

This is because you can invest the $12,000 so that it will accumulate to more than $12,000 at the end of 2 years. Another way of looking at this is to say that because of the time value of money, you would take an amount less than $12,000 if you could receive it today, instead of $12,000 in 2years. The amount you would be willing to accept depends on the interest rate or the rate of return you receive.

The formula used to calculate the present value of a single amount is:

PV = FV/(1 + i) n where PV = present value, FV = future value, i = decimalized interest rate, and n = number of periods.

Illustration 1:

Suppose a company expects to receive $8,000 after 5 years. Calculate the present value of this sum if the current market interest rate is 12% and the interest is compounded annually.

Solution:

The way to solve this is to apply the above present value formula. In this example, the number of periods (n) is 5 and the interest rate (i) is 12%. Therefore, the present value (PV) is calculated as follows:

$PV = FV \times 1 / (1+i)^n$

$= 8,000 \times 1 / (1+12\%)^5$

$= 8,000 \times 1 / (1+0.12)^5$

$= 8,000 \times 1 / (1.12)^5$

$= 8,000 \times 1 / 1.7623$

$= 8,000 \times 0.5674$

PV = $4,540

Illustration 2: What is the present value of $1,000 received in two years if the interest rate is?

(a) 12% per year discounted annually.

(b) 12% per year discounted semi-annually.

(c) 12% per year discounted daily

Solution:

(a) 12% per year discounted annually.

$=1,000 / (1 + 0.12)^2$

= $797.19

(b) 12% per year discounted semi-annually.

$= 1,000 / (1 + 0.12/2)^{2*2}$

= $792.09

(c) 12% per year discounted daily

$= 1,000 / (1 + 0.12/365)^{2*365}$

= $786.66

Illustration 3: $7,000 for 10 years from now at 7% is worth how much today?

Solution:

$7,000 / (1 + 0.07)^{10}$

= $3,558.45

Illustration 4: What is the present value of $84,253 to be received or paid in 5 years discounted at 11% by table and factor formula?

Solution:

= 84,253 (PVIF 11%, 5)

PV = 84,253 (0.5935)

Illustration 5:

Mr. Nadeem owes a total of $3,060 which includes 12% interest for the three years he borrowed the money. How much did he originally borrow?

Solution:

$= 3{,}060 / (1 + 0.12)^{3}$

Illustration 6:

If Ramesh want $2,000 three years from now and the compounded interest rate is 8%, how much should he invest today?

Solution:

$=2{,}000 / (1 + 0.08)^{3}$

= **$1,587.66**

Illustration 7:

What is the present value of an offer of $14,000 two years from now if the opportunity cost of capital (discount rate) is 17% per year discounted annually?

Solution:

$=14{,}000 / (1 + 0.17)^{2}$

= **$10,227.19**

Illustration 8:

If you invested $50,000 at one point in time and received back $80,000 ten years later, what annual interest (or growth) rate (compounded annually) would you have obtained?

Solution:

$= (80{,}000/50{,}000)^{(1/10)} – 1$

= **4.81%**

Illustration 9:

How much would you have to deposit today to have $10,000 in five years at 6% interest discounted quarterly?

Solution:

$= 10{,}000 / (1 + 0.06 / 4)^{5 * 4}$

= **$7,424.46**

Illustration 10:

What is the present value of an offer of $15,000 one year from now if the opportunity cost of capital (discount rate) is 12% per year nominal annual rate compounded monthly?

Solution:

$= 15{,}000 / (1 + 0.12/12)^{1*12}$

= **$13,311.74**

Illustration 11:

Calculate the present value of each cash flow using a discount rate of 7%. Which do you most prefer most?

S. No	***Cash Flows***	***Solution***
1	Cash flow A: receive $60 today and then receive $60 in four years	PV of A = 60+60*1.07 -4 = $105.77
2	Cash flow B: receive $12 every year, forever, starting today.	12+12/0.07= $183.43
3	Cash flow C: pay $50 every year for five years, with the first payment being next year, and then subsequently receive $30 every year for 20 years.	-50/0.07*(1-1.07-5)+30/0.07*(1-1.07-20)*1.07-5 = $21.59
4	Cash flow D: receive $9 every other year, forever, with the first payment being next year.	9/(1.072-1)*1.07 = $66.46

Present Value of an Uneven Series

In financial analysis we often witness uneven cash flow streams. For instance, the cash flow stream associated with a capital investment project is typically uneven. Likewise, the dividend stream associated with an equity share is usually uneven and perhaps growing.

The present value of a cash flow stream- uneven or even may be calculated with the help of the following formula:

Following is the formula to calculate the PV of uneven cash flows:

		CF_0		CF_1		CF_2				CF_N
PV	=	——	+	——	+	——	+	——	+	——
		$(1 + r)^0$		$(1 + r)^1$		$(1 + r)^2$				$(1 + r)^N$

In simple words, we can put the above formula as:

PV = Sum of CF_n/(1 + r)^n

In the above formula,

In is the number of years, CF_n is the cash flow for the year, and n and r is the discount rate for the year. The discount rate is generally the opportunity rate or interest that an asset could generate elsewhere.

The following table explains the present value of an uneven cash flow stream

Year	***Cash Flow (Rs.)***	***PVIF*** $_{12\%,n}$	***Present Value of Individual Cash Flow***
1	1,000	0.893	893
2	2,000	0.797	1,594

3	2,000	0.712	1,424
4	3,000	0.636	1,908
5	3,000	0.567	1,701
6	4,000	0.507	2,028
7	4,000	0.452	1,808
8	5,000	0.404	2,020
Present Value of the Cash Flow Stream			13,376

Illustration 12:

A project generates the following cash flows;

Beginning of years:

1 – ($100,000) (contractors' fees)

2 – ($200,000) (contractors' fees)

3 – ($200,000) (contractors' fees)

End of Year 3 : $1,000,000 (sales)

Calculate the NPV of the project using a risk discount rate of 20% per year.

Solution:

NPV = 100,000 – 200,000 $(1+ 0.2)^{-1}$ – 200,000$(1 + 0.2)^{-2}$ +1000,000$(1+ 0.2)^{-3}$

= -100,000 – 166,667 – 138,889 + 578,704

= **$173,148.**

Present Value of an Annuity

The present value of an annuity is the current value of future payments from an annuity, given a specified rate of return, or discount rate. The higher the discount rate, the lower the present value of the annuity.

Key points-

- The present value of an annuity refers to how much money would be needed today to fund a series of future annuity payments.
- Because of the time value of money, a sum of money received today is worth more than the same sum at a future date.
- One can use a present value calculation to determine whether he will receive more money by taking a lump sum now or an annuity spread out over a number of years.

The formula to compute Present Value of an Annuity is as under:

$$P = PMT \times \frac{1 - (1 / (1 + r)^n)}{r}$$

where:

P= Present value of an annuity stream.

PMT = Monetary value of each annuity payment.

r = Interest rate (also known as discount rate).

n = Number of periods in which payments will be made.

Illustration 13:

Assume a person has the opportunity to receive an ordinary annuity that pays $50,000 per year for the next 25 years, with a 6% discount rate, or take a $650,000 lump-sum payment. Which is the better option? Using the above formula, the present value of the annuity is:

$$\text{Present Value} = \$50{,}000 \times \frac{1 - (1 / (1+0.06)^{25})}{0.06}$$

= **$639,168**

Applications of Present Value of an Annuity

The present value annuity formula can be applied in a variety of contexts. Its important applications are as under:

a) *How much can you borrow for a car:* After reviewing your budget, you have ascertained that you can afford to pay Rs.12,000 per month for 3 years toward a new car. You call a finance company and learn that the current rate of interest on car finance is 1.5 percent per month for 36 months. How much you borrow?

To determine how much you can borrow, we have to calculate the present value of Rs.12,000 per month for 36 months at 1.5 percent per month.

Since the loan payments are an ordinary annuity, the present value interest factor annuity is –

$$PVIFA_{r,n} = \frac{1 \quad \frac{1}{(1+r)^n}}{r}$$

$$= \frac{1 - \frac{1}{(1.015)^{36}}}{0.15} = 27.70$$

Hence the present value of 36 payments of Rs.12,000 each is:

Present value = Rs.12,000 x 27.70 = Rs.332,400

You can, therefore, borrow Rs.332,400 to buy the car.

b) *Period of Loan Amortisation:* You want to borrow Rs.1,080,000 to buy a flat. You approach a housing finance company which charges 12.5 percent interest. You can pay Rs.180,000 per year toward loan amortisation. What should be the maturity period of the loan?

The present value of annuity of Rs.180,000 is set equal to Rs. 1,080,000.

$180{,}000 \times PVIFA_{n,r} = 1{,}080{,}000$

$180{,}000 \times PVIFA_{n=?,\, r = 12.5\%} = 1{,}080{,}000$

$$180{,}000 \left[\frac{1 \quad \frac{1}{(1.125)^n}}{0.125} \right]$$

= 1,080,000

Given this equality the value of n is calculated as follows:

$$\frac{1 \quad \frac{1}{(1.125)^n}}{0.125}$$

$$= \frac{1{,}080{,}000}{180{,}000} = 6$$

$$= \frac{1}{(1.125)^n} = 0.25$$

$= 1.125^n = 4$

$$n = \frac{0.6021}{0.0512} = 11.76 \text{ years}$$

You can perhaps request for a maturity of 12 years.

c) *Determining the Loan Amortisation Schedule:* Most loans are repaid in equal periodic instalments (monthly, quarterly, or annually), which cover interest and principal repayment. Such loans are referred to as amortised loans.

For an amortised loan we would like to know i) the periodic instalment payment and ii) the loan amortisation schedule showing the break up of the periodic instalments payments between the interest component and the principal repayment component. To illustrate how these are calculated, let us look at an example.

Suppose a firm borrows Rs.1,000,000 at an interest rate of 15 percent and the loan is to be repaid in 5 equal instalments payable at the end of each of the next 5 years. The annual instalment payment A is obtained by solving the following equation.

Loan amount = A x $PVIFA_{n= 5,\, r = 15\%}$

1,000,000 = A x 3.3522

Hence A = 298,312

The amortisation schedule is provided below. The interest component is the largest for year 1 and progressively declines as the outstanding loan amount decreases.

Loan Amortisation Schedule

Year	*Beginning Amount (1)*	*Annual Installment (2)*	*Interest (1) x 0.15 =(3)*	*Principal Repayment (2) – (3) = (4)*	*Remaining Balance (1) – (4) = (5)*
1	1,000,000	298,312	150,000	148,312	851,688
2	851,688	298,312	127,753	170,559	681,129
3	681,129	298,312	102,169	196,143	485,986
4	485,986	298,312	72,482	225,564	259,422
5	259,422	298,312	38,913	259,399	23

d) *Determining the Periodic Withdrawal:* Suppose your father deposits Rs.300,000 on retirement in a bank which pays 10 percent annual interest. How much can be withdrawn annually for a period of 10 years?

$$A = \text{Rs.}300{,}000 \times \frac{1}{PVIFA_{10\%,\,10}}$$

$$= \text{Rs.}300{,}000 \times \frac{1}{6.145}$$

= Rs.48,819.

e) *Finding the Interest Rate:* let us assume that someone offers you the following financial contract: If you deposit Rs.10,000 with him he promises to pay Rs.2,500 annually for 6 years. What interest rate do you earn on this deposit? The interest rate may be calculated in two steps:

Step 1: Find the $PVIFA_{r,6}$ for this contract by dividing Rs.10,000 by Rs.2,500

$$PVIFA_{r,6} = \frac{\text{Rs.}10{,}000}{\text{Rs.}2{,}500} = 4\%$$

Step 2: Look at the PVIFA table and read the row corresponding to 6 years until you find a value close to 4.000. Doing so, you find that

$PVIFA_{12\%,\,6}$ is 4.111 and $PVIFA_{14\%,6}$ is 3.889

Since 4.000 lies in the middle of these values the interest rate lies (approximately) in the middle. So, the interest rate is 13 percent.

f) *Valuing an Infrequent Annuity:* Rajan will receive an annuity of Rs.50,000, payable once every two years. The payments will stretch out over 30 years. The first payment will be received at the end of two years. If the annual interest rate is 8 percent, what is the present value of the annuity?

The interest rate over a two-year period is, (1.08) x (1.08) – 1 =16.64 percent.

This means that Rs.100 invested over two years will yield Rs.116.64.

Here, present value of Rs.50,000 annuity over 15 periods needs to be calculated, with an interest rate of 16.64 percent per period. This works out to:

Rs.50,000 $[\{ 1 – (1/1.1664)^{15} \}] / 0.1664$ = Rs.270,620

g) *Equating Present Value of Two Annuities:* Raj wants to save for the college education of his son, Deepak.

Ravi estimates that the college education expenses will be rupees one million per year for four years when his son reaches college in 16 years – the expenses will be payable at the beginning of the years. He expects the annual interest rate of 8 percent over the next two decades. How much money should he deposit

Illustration 14:

Issac has just won the lottery and decides to take the 20 year annuity option. The lottery commission invests his winnings in an account that pays 4.8% interest, compounded annually. Each year for those 20 years, Tom receives a check from the lottery commission for $250,000. What is the present value of Tom's winnings? (Notice that this would be the amount that Tom would get if he chose the lump-sum option). What is the total amount of money that Tom gets over the 20 year period?

Solution:

This is clearly an annuity question since it says so in the problem. We are told what the payments are for the annuity, and asked to find the present value, so we use the present value formula for an annuity:

$$PV = PMT \times \frac{1-(1+i)^{-n}}{i}$$

Since this annuity is compounded annually (and the payments are made annually), (meaning and), and we get

$$PV = 250000 \times \frac{1-(1+0.048)^{-20}}{\mathbf{0.048}}$$

= **$3169070.90**

Illustration 15: John has just received an inheritance of $400,000 and would like to be able to make monthly withdrawals over the next 15 years. She decides on an annuity that pays 6.7%, compounded monthly. How much will her monthly payments be in order to draw the account down to zero at the end of 15 years?

Solution:

Since John will be making periodic withdrawals from an account, this is an annuity question. She would like to know how much each withdrawal will be so that the entire inheritance will be gone after 15 years. We use the payment formula for an annuity to find out how much each withdrawal (payment) will be:

$$PMT = PV \times \frac{i}{1-(1+i)^{-n}}$$

$$= 400{,}000 \times \frac{0.067 / 12}{1-(1+i)^{-12*15}} = \mathbf{\$3528.56}$$

Thus, each withdrawal will be $3,528.56. At the end of the 15 years, nothing will be left.

Illustration 16:

Amar is working in a tire factory that offers a pension in the form of an annuity that pays 5% annual interest, compounded monthly. He wants to work for 30 years and then have a retirement income of $4000 per month for 25 years. How much do he and his employer together have to deposit per month into the pension fund to accomplish this?

Solution:

This problem is probably the most realistic, and most closely matches what a typical person will do in his or her life (save money during their working life, then spend that money during retirement). The only thing we know is what Amar wants to have during retirement: $4,000 per month for 25 years.

Since this is money he will be withdrawing from an account, it is an annuity. We would first like to know how much money he needs in order to be able to make these monthly withdrawals for 25 years. Thus, we need the present value of an annuity:

$$PV = PMT \times \frac{1-(1+i)^{-n}}{i}$$

$$= 4000 \times \frac{1-(1+0.05/12)^{-12*25}}{0.05/12}$$

= $684,240.19

Thus, Amar will need $684,240.19 to fund his retirement annuity.

Illustration 16 (A):

Rebecea has set up a savings account with her bank and will be paying $350 a month into the account for the next five years. The annual interest rate is 3% and the annual growth rate is 2%. How can Rebecca work out the present value of these payments?

Solution: Since the interest in this example is applied annually, the number of periods (n) will be 5, and the total annual payment is $350 x 12 = $4,200.

If the interest rate was applied monthly, we would take the annual interest rates and divide them by 12 to get a monthly discount rate (i) of 0.0025% and a monthly growth rate (g) of 0.0017%, using a total number of periods (n) of 60.

$$PV = \$4,200 \text{ X } \frac{(1-(1+2\%)^{5} \times (1+3\%)^{-5}}{2\% - 3\%} = \$19,996.28$$

Now, what if Rebecea's bank did pay the interest monthly instead of annually? In that case, the formula would look like this:

$$PV = \$350 \times \frac{(1-(1+0.0017\%)^{60} \times (1+0.0025\%)^{-60})}{0.0017\% - 0.0025\%} = \$20,994.52$$

It can be seen that the PV of the annuity is growing faster because the payments are compounding 12 times a year at the 2% growth rate instead of just once a year with annual interest.

Illustration 16(B):

Mr. Z is looking ahead to his retirement and want to be able to retire at 70 and hope to live to 95 and make $3200 a month from an account compounding monthly at 4.5%. He is currently 27 and going to deposit

$1000 at the beginning of each quarter until he is 70 in an account that pays 8.5% and is compounded quarterly. Will he have enough to make it happen and by how much amount he is having surplus or deficit?

Solution: Find the amount Mr.Z need to support those requirements from age 70 to 95.

$$PV = 3200 \frac{1 - (1 + 0.045/12)^{-12\ (25)}}{0.045/12}$$

PV = $575713.03 is needed by Mr. Z to support himself from 70-95 years old.

Present Value of Perpetuity

Perpetuity can be defined as the income stream that the individual gets for an infinite time period and its present value is arrived at by discounting the identical cash flows with the discounting rate. Here the cash flows are infinite but its present value amounts to a limited value.

Perpetuity is a series of cash flows that have an infinite life, and such an income stream grows with a proportionate rate. The cash flows should be identical.

The formula is basically derived from the dividend growth model. The formula attempts to determine the terminal value of the identical cash flows. Therefore, the present value of the cash flows at basic expression can be derived as follows: –

Present value = D / (1+r) + D x (1 + g) / (1 + r) ^2 + D / (1+r) + D x (1 + g) ^2 / (1 + r) ^3.........

PV of Perpetuity = ICF / r

Here,

- The identical cash flows are regarded as the CF.
- The interest rate or the discounting rate is expressed as r.

If the perpetuity grows by a constant growth rate, then it would be expressed as described below: –

PV of Perpetuity = ICF / (r – g)

Here,

The identical cash flows are regarded as the CF.

The interest rate or the discounting rate is expressed as r.

The growth rate is expressed as g.

Uses of Present Value of Perpetuity

- Perpetuity is normally utilized in preferred stocks.
- The preferred stocks tend to provide fixed dividends throughout the company life cycle.
- Since the perpetuity is an infinite amount, its present value helps in arriving at a value that has a limited amount.
- The perpetuity has its applications in real estate as well.
- If the real estate provides a sustainable income stream, then its present value is derived using the relationship of the present value of a perpetuity.

- Additionally, the PV of the perpetuity forms the basis for several endowment schemes and retirement planning.
- Endowment schemes are financial protection plans that provide financial protections as well as cater to a comprehensive saving plan.
- Such schemes, if planned properly, can deliver a fixed income stream for infinite tenure.

Illustration 17:

Magnificent Limited pays $2 in dividends annually and estimates that they will pay the dividends indefinitely. How much are investors willing to pay for the dividend with a required rate of return of 5%?

Solution: PV = 2/5% = $40

An investor will consider investing in the company if the stock price is $40 or less.

Future Value of a Single Amount

The value of a current single amount taken to a future date at a specified interest rate is called the future value of a single amount.

In this case, "future value" means the amount to which the investment will grow at a future date if interest is compounded. The single amount refers to a lump sum invested at the beginning of a period (e.g., year 1) and left intact for all periods.

Formula and Calculation of Future Value-

FV=I×(1+(R×T))

where:

I=Investment amount

R=Interest rate

T=Number of years

To explain the concept of the future value of a single amount, let's start with the table below.

Year	*Principal amount at beginning of the year ($)*	*Annual interest income @12% ($)*	*Accumulated at end of the year ($)*
1	10,000.00	1,200.00	11,200.00
2	11,200.00	1,344.00	12,544.00
3	12,544.00	1,505.00	14,049.28

In the above table, we see what the future amount of $10,000 invested at 12% annual interest for three years would be given a certain compounding pattern. This is an example of determining the future value of a single amount.

There were no additional investments or interest withdrawals. These future value or compound interest calculations are important in many personal and business financial decisions.

Future value (FV) is the value of a current asset at a future date based on an assumed rate of growth. The future value is important to investors and financial planners, as they use it to estimate how much an investment made today will be worth in the future.

Knowing the future value enables investors to make sound investment decisions based on their anticipated needs. However, external economic factors, such as inflation, can adversely affect the future value of the asset by eroding its value.

Illustration 18:

You are scheduled to receive Rs.13,000 in two years. When you receive it, you will invest it for six more years at 8 percent per year. How much will you have in eight years?

Solution: The amount that will be received in eight years will be –

$= 13{,}000\ (1 + 0.08)^{6}$

= Rs. 20,629.37

Illustration 19:

You have Rs.9,000 to deposit. Jupiter Bank offers 12 percent per year compounded monthly, while Saturn Bank offers 12 percent but will only compound annually. How much will your investment be worth in 10 years at each bank?

Solution:

Jupiter Bank

$9{,}000\ (1 + 0.12/12)^{10 * 12}$

= Rs.29,703.48

Saturn Bank

$9{,}000\ (1 + 0.12)^{10}$

= Rs.27,952.63

Variance = **Rs.1,750.85**

Illustration 20:

What is the future value of Rs. 26 invested for 32 years at an average rate of return of 7%?

Solution:

$FV = 26\ (1.07)^{32}$

= Rs.226.60

Illustration 21:

Find the future value of Rs.100,000 for 15 years. The current five-year rate is 6%. Rates for the second and third five-year periods and expected to be 6.5% and 7.5%, respectively.

Solution:

FV = 100,000 $(1.06)^5(1.065)^5(1.075)^5$

FV = 100,000 (1.3382) (1.37009) (1.43563)

FV = 100,000 (2.6322)

FV = Rs.263,220

Illustration 22:

If farm land is currently worth Rs. 1,750 per acre and is expected to increase in value at a rate of 5 percent annually, what will it be worth in 5 years? In 10 years? In 20 years by factor formula and table?

Solution:

i) In 5 years

= Rs. 1,750 x 1.2763

= **Rs.2,233.53**

ii) In 10 years

= Rs. 1,750 x 1.6289

= **Rs. 2,850.58**

iii) In 20 years

= Rs.1,750 x 2.6533

= **Rs.4,643.28**

Illustration 23:

what will be the future value at the end of the 5 years of $1,000 paying a 5% rate of interest?

Solution:

= 1000 $(1+.10)^5$

FV= 1620

Illustration 24:

If a person deposits $100 at the end of the first year, $200 at the end of the second year, and $250 at the end of the third year in a bank, what will be his future value if the interest rate is 10%?

Solution:

= 100 $(1+.10)^3$-1 + 200 $(1+.10)^{3-}$2 +250 $(1+.10)^3$-3

= 121+220+250

FV= 591

Illustration 25:

You decide to put $12,000 in a money market fund that pays interest at the annual rate of 8.4%, compounding it monthly. You plan to take the money out after one year and pay the income tax on the interest earned. You are in the 15% tax bracket. Find the total amount available to you after taxes.

Solution: The monthly interest rate is .084/12 = .007. Using it as the growth rate, the future value of money after twelve months is:

FV = 12000(1.007)12 = $13,047.73

The interest earned = 13,047.73 – 12,000 = $1047.73. You have to pay 15% tax on this amount. Thus after paying taxes, it becomes =1047.73(1 – .15) = $890.57.

Total amount available after 12 months = 12,000 + 890.57 = $12,890.57.

Illustration 26:

You have borrowed $850 from your sister and you have promised to pay her $1000 after 3 years. With annual compounding, find the implied rate of interest for this loan.

Solution: The future value of the loaned money is FV = $1000, while its present value is PV = $850. The time for compounding is n = 3 years. The interest rate r is unknown.

Using $FV = PV (1 + r)^n$

We get $1000 = 850(1 + r)^3$

or, $(1000/850)^{1/3} = 1 + r$

or, 1 + r = 1.0556672

which gives r = 0.0557 = 5.57%

Illustration 27:

You have borrowed $10,000 from a bank with the understanding that you will pay it off with a lump sum of $12,000 after 2 years. Find the annual rate of interest on this loan.

Solution: Here the future value is $12,000, present value $10,000, and n = 2. Use

$FV = PV (1 + r)^n$

This gives $12,000 = 10,000 (1 + r)^2$

Or, r = 12,000 10,000 − 1 = .09545 = 9.545%

Illustration 28:

Global Banking Corporation offers two types of certificates of deposit, each requiring a deposit of $10,000. The first one pays $11,271.60 after 24 months, and the second one pays $12,139.47 after 36 months. Find their monthly-compounded rate of return.

Solution: Using $FV = PV (1 + r)^n$

We get for the first CD,

$11,271.60 = 10,000(1 + R1)^{24}$

Solving for R_1, we get

$$R_1 = \left(\frac{11,271.60}{10,000}\right)^{1/24}$$

Similarly working on the second CD, we get

$$R_2 = \left(\frac{11{,}271.60}{10{,}000}\right)^{1/24}$$

The first certificate gives a return of .5%, and the second one .54% per month. The second one is higher because the investor has to tie up the money for a longer period

Illustration 29:

A bank account pays 5.5% annual interest, compounded monthly. How long will it take the money to double in this account?

Solution: If the present value is $1, its future value is $2. The bank is compounding monthly, thus the interest rate is 5.5/12 percent per month.

$FV = PV (1 + r)^n$

we get $2 = 1(1 + .055/12)^n$

Taking logarithms of both sides, ln 2 = n ln(1.0045833),

or, n = ln(2) / ln(1.0045833)

= 151.58 months = approximately, 12 years and 8 months

Future Value of an Annuity

The future value of an annuity is the value of a group of recurring payments at a certain date in the future, assuming a particular rate of return, or discount rate. The higher the discount rate, the greater the annuity's future value.

Key Features:

- The future value of an annuity is a way of calculating how much money a series of payments will be worth at a certain point in the future.
- By contrast, the present value of an annuity measures how much money will be required to produce a series of future payments.
- In an ordinary annuity, payments are made at the end of each agreed-upon period. In an annuity due, payments are made at the beginning of each period.

The formula for the future value of an ordinary annuity is as follows. (An ordinary annuity pays interest at the end of a particular period, rather than at the beginning, as is the case with an annuity due.)

$$P = PMT \times \frac{(1 + r)^n}{r}$$

where:

P = Future value of an annuity stream

PMT = Dollar amount of each annuity payment

r = Interest rate (also known as discount rate)

n = Number of periods in which payments will be made

Illustration 30:

Assume someone decides to invest $125,000 per year for the next five years in an annuity they expect to compound at 8% per year. What will be the expected future value of this payment stream?

Solution: Future Value = $\$125{,}000 \times \frac{((1 + 0.08)^5 - 1)}{0.08} = \$733{,}325$

With an annuity due, where payments are made at the beginning of each period, the formula is slightly different. To find the future value of an annuity due, simply multiply the formula above by a factor of (1 + r). So:

$$P = PMT \times \frac{((1 + r)^n - 1)}{r} \times (1 + r)$$

If the same example as above were an annuity due, its future value would be calculated as follows:

$$\text{Future Value} = \$125{,}000 \times \frac{((1 + 0.08)^5 - 1)}{0.08} \times (1 + 0.08)$$

= $791,991.

Illustration 31:

John deposits money into his savings account at the beginning of each year, depending on the returns of the business. He deposits $1000 in the first year, $2000 in the second year, $5000 in the third, and $7000 in the fourth year. The account credits interest at an annual interest rate of 7%. What is the closest value of the accumulated money in the savings account at the beginning of year 4?

Solution: The future value of the unequal payments is the sum of individual accumulations:

$= 1000(1.07)^3 + 2000(1.07)^2 + 5000(1.07)^1 + 7000(1.07)^0 = \$16{,}975.38$

Illustration 32:

Suppose Arjun invest $2000 per year in a stock index fund, which earns 9% per year, for the next ten years, what would be the closest value of the accumulated value of the investment upon payment of the last instalment?

Solution: From the information given in the question:

A=2000

N=10

r=9%

So that:

$$FV_N = A \frac{(1 + r)^N - 1}{r}$$

$$= 2000 \frac{(1 + r)^{10} - 1}{0.09}$$

= $ 30,385.8594

Illustration 33:

An individual makes rental payments of $1,200 per month and wants to know the present value of their annual rentals over a 12-month period. The payments are made at the start of each month. The current interest rate is 8% per annum.

Solution: PV

$$= \$1,200 \times \frac{1-(1+(0.08/12))^{-12}}{(0.08/12)} \times (1+(0.08/12))$$

FV of investment = $1200 x 11.57

FV of investment = **$13,886.90**

Illustration 34:

A company wants to invest $3,500 every six months for four years to purchase a delivery truck. The investment will be compounded at an annual interest rate of 12% per annum. The initial investment will be made now, and thereafter, at the beginning of every six months. What is the future value of the cash flow payments?

Solution:

$$\text{FV of the investment} = \frac{\$3500 \times (1+(0.12/2))^{2\times 4} - 1}{(0.12/2)} \times (1+(0.12/2))$$

FV of the Investment = $3,500 x 10.49

FV of the Investment = **$36,719.61**

Annuity Due vs. Ordinary Annuity

1. Payments: The major difference between annuity due and the more popular ordinary annuity is that payments for an ordinary annuity are made at the end of the period, as opposed to annuity due payments made at the start of each period/interval. Ordinary annuity payments include loan repayments, mortgage payments, bond interest payments, and dividend payments.

2. Present value: Another difference is that the present value of an annuity due is higher than one for an ordinary annuity. It is a result of the time value of money principle, as annuity due payments are received earlier.

Hence, if you are set to make ordinary annuity payments, you will benefit from getting an ordinary annuity by holding onto your money longer (for the interval). Conversely, if you are set to receive annuity due payments, you will benefit, as you will be able to receive your money (value) sooner. In any annuity due, each payment is discounted one less period in contrast to a similar ordinary annuity.

Doubling Period

Investors commonly ask the question: How long would it take to double the amount at a given rate of interest? To ascertain the doubling period, two rules are followed, i.e., Rule of 72 and Rule of 69.

i) *Rule of 72:* The Rule of 72 dates back to 1494 when Luca Pacioli referenced the rule in his comprehensive mathematics book called Summa de Arithmetica. Pacioli makes no derivation or explanation of why the rule may work, so some suspect the rule pre-dates Pacioli's novel.

According to this rule of thumb the doubling period is obtained by dividing 72 by the interest rate. For example, if the interest rate is 8 percent, the doubling period is nearly 9 years (72/8). Likewise, if the interest rate is 4 percent the doubling period is nearly 18 years (72/4). Though somewhat crude, it is a handy and useful rule of thumb.

The key takeaways of the Rule of 72 are as under:

- The Rule of 72 is a simplified formula that calculates how long it'll take for an investment to double in value, based on its rate of return.
- The Rule of 72 applies to compounded interest rates and is reasonably accurate for interest rates that fall in the range of 6% and 10%.
- The Rule of 72 can be applied to anything that increases exponentially, such as GDP or inflation; it can also indicate the long-term effect of annual fees on an investment's growth.
- This estimation tool can also be used to estimate the rate of return needed for an investment to double given an investment period.
- For different situations, it's often better to use the Rule of 69, Rule of 70, or Rule of 73.

ii) *Rule of 69:* If you are inclined to a slightly more involved calculation, a more accurate rule of thumb is the rule of 69. According to this rule of thumb, the doubling period is equal to:

$$0.35 + \frac{69}{\text{Interest Rate}}$$

The following are the benefits of the rule of 69.

- It assumes that the interest is continuously compounding. In fact, it is true to think in the case of equity valuation that is compounding on an instant basis.
- It provides the answer very close to the answer obtained by using a financial calculator.
- It is even considered as the thumb rule of the investment return generating a cycle.
- Easy to calculate the time required.
- Even the retail investor or a non-finance person can easily determine the result.
- Can be used by any person without understanding the pure logic.
- Faster decision making and improve thought process.

The following are the limitations of the rule of 69.

- Difficult to explain the logic behind the number 69.
- Rule 69 does not apply to everything. Only the security like equity, which is compounding every minute, can provide the exact value (Rule 72 can Be help in those cases)
- If the rate is too less like 2/3 % Per annum, than the result is not very accurate. Generally, the higher rate is well captured by this formula.
- Projects with a heavy investment need specially designed spreadsheets because a minuscule difference in time and rate of interest value can create a difference of millions.
- Difficult to absorb the value derived because of non-transparency of value derivation.
- This rule covers those instruments which compounds continuously like equity shares, but it ignores the dividend component which is also received by the equity holder, so overall the share did not increase by an exact multiple of 2, but the dividend amount makes the value of it.

As an illustration of this rule of thumb, the doubling period is calculated for two interest rates, 10 percent and 15 percent.

Interest Rate	***Doubling Period***
10 percent	0.35 + 69 /10 = 7.25 years
15 percent	0.35 + 69 /15 = 4.95 years

iii) *Rule of 70:* The rule of 70 is used to determine the number of years it takes for a variable to double by dividing the number 70 by the variable's growth rate. The rule of 70 is generally used to determine how long it would take for an investment to double given the annual rate of return.

For example, assume an investor invests $10,000 at a 10% fixed annual interest rate. He wants to estimate the number of years it would take for his investment to grow to $20,000. He uses the rule of 70 and determines it would take approximately seven (70/10) years for his investment to double.

Illustration 35:

Due to the large capital needed to establish a factory and warehouse for coffee machines, Akshay have turned to private investors to fund the expenditure. He met with Jacob, who is a high net-worth individual willing to contribute $1,000,000 to Akshay's company.

However, Jacob is only willing to contribute the said amount on the presumption that he will get a 12% annual rate of return on his investment, compounded yearly. He wants to know how long it will take for his investment in Akshay's company to double in value.

Solution:

Using the Rule of 72-

Doubling time (number of years) = 72 /12% = 6 years.

It will take approximately six years for Jacob's investment to double in value.

LESSON ROUND-UP

- The time value of money means that a sum of money is worth more now than the same sum of money in the future.
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- The time value of money means that a sum of money is worth more now than the same sum of money in the future.
- The most fundamental formula for the time value of money takes into account the following: the future value of money, the present value of money, the interest rate, the number of compounding periods per year, and the number of years.

- Money has time value because of the following reasons:
  - i) Risk and Uncertainty. Future is always uncertain and risky.
  - ii) Inflation: In an inflationary economy, the money received today, has more purchasing power than the money to be received in future.
  - iii) Consumption.
  - iv) Investment opportunities.
- Time value of money is important because it helps investors and people saving for retirement determine how to get the most out of their dollars. This concept is fundamental to financial literacy and applies to your savings, investments and purchasing power.

GLOSSARY

Annuity : An annuity is a series of equal cash flows paid at equal time intervals for a finite number of periods. A lease that calls for payments of $1000 each month for a year would be referred to as a "12-period, $1000 annuity." Note that, strictly speaking, in order for a series of cash flows to be considered an annuity, each cash flow must be identical and the amount of time between each cash flow must be the same in all cases. There are two types of annuities that vary only in the timing of the first cash flow:

- Regular Annuity – The first payment is made one period in the future (at period 1).
- Annuity Due – The first payment is made immediately (at period 0).

Graduated Annuity : A graduated annuity (also called a growing annuity) is a series of cash flows that increases over time at a constant rate for a finite number of periods. A common example of a graduated annuity would be a lottery payout.

A lottery winner (e.g., Powerball) may opt to receive their winnings as a series of 30 annual payments (the first payment is immediate, and there are 29 additional annual payments). In the case of Powerball, each payment will be 4% greater than the previous payment. Note that, strictly speaking, a graduated annuity requires that the growth rate of the payments be constant for the life of the annuity.

Lump Sum: A lump sum is a single cash flow. For example, an investment that is expected to pay $100 one year from now would have a "lump sum payment" of $100.

Perpetuity: A perpetuity is simply a type of annuity that has an infinite life. In other words, it is a "perpetual annuity."

Uneven Cash Flow Stream: Any series of cash flows that doesn't conform to the definition of an annuity is considered to be an uneven cash flow stream.

Amortization Schedule: An amortization schedule is a table that shows each loan payment over the life of a loan, and a breakdown of the amount of interest and principal paid. Typically, it will also show the remaining balance after each payment has been made. Please see my tutorial on how to create an amortization schedule in Excel for more information.

Cash Flow Sign Convention: This convention, used by financial calculators and spreadsheet functions, specifies that the sign (i.e., positive or negative numbers) indicates the direction of the cash flow. Cash inflows are entered as positive numbers, and cash outflows are entered as negative numbers. Failure to properly adhere to this convention will usually result in incorrect answers from your calculator or spreadsheet. Please note that whether a cash flow is an inflow (+) or outflow (-) depends on the part that you play in a transaction. For example, loan payments are an outflow (-) for the borrower, but an inflow (+) for the lender.

Principle of Value Additivity: This fundamental principle states that the present value (future value) of a series of cash flows is the sum of the present value (future value) of each of the individual cash flows. For example, we can calculate the present value of an annuity by using a single formula, or by calculating the present value of each individual cash flow and then adding them together.

This principle is very often useful for simplifying the calculation of the present or future value of uneven cash flow streams, particularly if the cash flows follow some identifiable pattern (such as several consecutive annuities).

Rule of 72: A simple rule that can be used to approximate how long it will take a given amount of money to double at a particular interest rate. It can also be used to determine the interest rate that is required to double your money in a particular amount of time. To determine how long it will take to double your money, simply divide 72 by the interest rate (in decimal form).

Compound Interest: This refers to the situation where, in future periods, interest is earned not only on the original principal amount, but also on the previously earned interest. This is a very powerful concept that means money can grow at an exponential rate.

Compounding Frequency: This refers to how often interest is credited to the account. Once interest is credited it becomes, in effect, principal. Note that the compounding frequency and the frequency of cash flows are not always the same. In that case, the interest rate is typically adjusted to an effective rate that is of the same periodicity as the cash flows.

Discount Rate: This is the interest rate that is used to convert between future values and present values. Note that the process of calculating present values is often referred to as "discounting" because present values are generally less than future values.

Frequency of Cash Flows: When using the cash flow functions, many financial calculators prompt you for both the cash flow (CFx) and then the frequency (Fx or #Times). The frequency is simple a shortcut to save both time and memory. If a cash flow occurs more than one time in a row, then you would enter the number of times that it occurs (in most cases, you will leave it at 1). The next cash flow that is entered will be the next different cash flow.

Future Value: This term refers to the value of a cash flow (or series of them) at some specific future time. Any cash flow that is scheduled to occur sometime later than today is referred to as a "future value." Literally translated, future value means "what will it be worth at some future point in time?"

TEST YOURSELF

1. Astral Limited 2020 sales were $100 million. If sales grow at 8% per year, how large will they be 10 years later, in 2030, in millions?
2. Suppose a U.S. government bond will pay $1,000 three years from now. If the going interest rate on 3-year government bonds is 4%, how much is the bond worth today?
3. You have a chance to buy an annuity that pays $1,000 at the end of each year for 5 years. You could earn 6% on your money in other investments with equal risk. What is the most you should pay for the annuity?
4. An investor deposited $10,000 in a savings account paying 5% converted quarterly. At the end of 5 years what is the value of the account?

5. A depositor planned to leave $2,000 in a savings account paying 5% converted semiannually for 5 years. However, at the end of 2 1/2 years the depositor had to withdraw $1,000. What amount will be in the account at the end of the original 5 year period?
6. Find the value of $1,000 invested at 8% for 10 years with interest compounded annually.
7. Find the amount of $6,000 invested at 12% for 5 years, compounded -- Annually, Semi-Annually, Quarterly, Monthly, Daily.
8. Find the present value of $5,000 due in 4 years if money is worth 4% compounded semi-annually.
9. What is the present value of a certificate of deposit with a maturity value of $1,000 due in 3 years, if money is worth 6% compounded semi-annually?
10. A person can buy a piece of property for $4500 cash OR for $2000 down and $3000 in 3 years. If money is earning 6% compounded semi-annually, which is the better purchase plan and by how much?
11. A piece of property can be purchased for $2850 cash OR for $3000 in 12 months. Which is the better plan if money is worth 7% compounded quarterly?
12. Find the amount of an annuity of $5,000 per year for 10 years at 6% and 7% with interest compounded annually.
13. What is the value of an annuity of $100 paid monthly for 6 years if money is worth 6% compounded monthly?
14. An investor wants to provide for a $3000 scholarship every year for 10 years. If the school can get a 5.5% return on its investment, how much money should the investor give now?
15. Wilson agrees to pay Smith $1000 each year for 5 years. If money is worth 7% what is the cash equivalent of this debt?
16. If money is worth 9% converted semi-annually, what is the present value of $145.50 due every 6 months for 2 years?
17. An investor makes a $2000 annual deposit into a mutual fund that produces a return of 12% annually for 3 years. How much will the investor have at the end of the three year term?
18. What is the annual yield on: a) a 3% account compounded monthly b) a 6 1/8% account compounded daily c) a 9% account compounded semi-annually.
19. If the present value (PV) of an investment is $10 million, and the amount is invested at a rate of return of 10% for one year, the future value will be equal to:
20. If $321 is invested at 2.5% interest compounded quarterly, calculate its value after 7 years.

LIST OF FURTHER READINGS

- Fundamentals of Financial Management by Dr. R.P. Rustagi, Taxmann.
- Financial Management: Theory and Practice by Dr Eugene F Brigham & C Micheal Ehrhardt.
- Fundamentals of Financial Management: Concise Edition by Brigham Houston.
- The Total Money Makeover: A Proven Plan for Financial Fitness by Dave Ramsey.
- Financial Management by I.M. Pandey.
- Financial Management: Theory and Practice, 10th Edition by Prasanna Chandra, McGraw-Hill.

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